THT 2 : 2(B)(d)(ii) | STAT 206 5 Mar 21 | DD extra OH

$\underset{\sim}{\theta} = (\theta_1, \dots, \theta_k)$

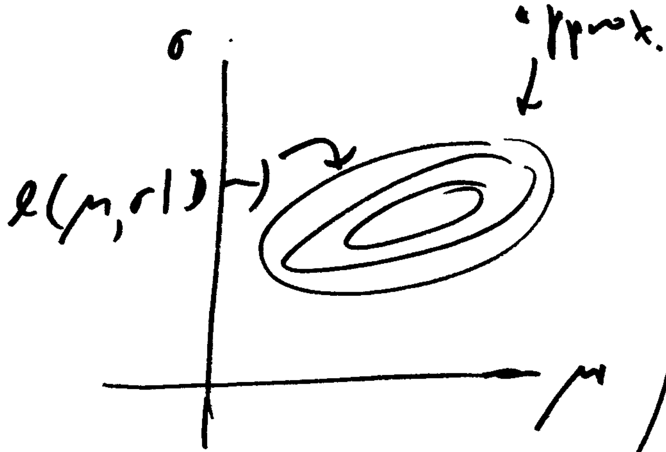
$\underset{\sim}{\eta} = (\eta_1, \dots, \eta_\ell)$

$\underset{\sim}{\eta} = (\mu, \sigma)$

$\ell = 2$

Bivariate Normal Dist.

$$\left(\hat{\underset{\sim}{\eta}} \,\middle|\, \underset{\sim}{D}\,[SM]\,\mathcal{B}\right) \,\dot\sim\, N_\ell\left[\underset{\sim}{\eta}, \Sigma_{\underset{\sim}{\eta}}\right]$$



$E(X)$ ← 1 r.v.

2 r.v.

$$E(\hat{\mu}_{MLE}, \hat{\sigma}_{MLE}) = \left[E(\hat{\mu}_{MLE}), E(\hat{\sigma}_{MLE})\right]$$

1 r.v.

$V(X)$

$$V(\hat{\mu}_{MLE}, \hat{\sigma}_{MLE}) = ?$$

$$\Sigma_{\hat{\mu},\hat{\sigma}} = \begin{array}{c|cc} & \hat{\mu} & \hat{\sigma} \\ \hline \hat{\mu} & C(\hat{\mu},\hat{\mu}) & C(\hat{\mu},\hat{\sigma}) \\ \hat{\sigma} & \underbrace{C(\hat{\sigma},\hat{\mu})}_{= C(\hat{\mu},\hat{\sigma})} & C(\hat{\sigma},\hat{\sigma}) \end{array}$$

covariance matrix

$$C(X, Y) = E(X \cdot Y) - [E(X) \cdot E(Y)]$$

$$C(X, X) = V(X)$$

$$\hat{\Sigma}_{(\hat\mu,\hat\sigma)} = \begin{bmatrix} \hat V(\hat\mu) & \hat C(\hat\mu,\hat\sigma) \\ \hat C(\hat\mu,\hat\sigma) & \hat V(\hat\sigma) \end{bmatrix}$$

for any X that is ~~the~~ an estimate of something,

$$SE(X) \doteq \sqrt{V(X)}$$

$$\hat{SE}(\hat\mu) = \sqrt{\hat V(\hat\mu)} = \sqrt{(1,1) \text{ entry in } \hat\Sigma_{(\hat\mu,\hat\sigma)}}$$

$(\ell \geq 2)$ $\hat\Sigma_{(\hat\mu,\hat\sigma)}$

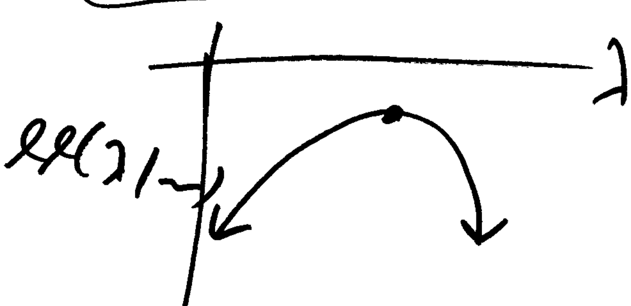
$(\ell = 1)$

$$\boxed{\hat V(\hat\lambda_{MLE})} = \left(\hat I(\hat\lambda_{MLE})\right)^{-1}$$

$$\hat I(\hat\lambda_{MLE}) = \left[-\frac{d^2}{d\lambda^2}\,\ell\ell(\lambda \mid D-)\right] \geq 0$$

observed information

$\lambda = \hat\lambda_{MLE}$



generalization of $\left[- \frac{\partial^2}{\partial \lambda^2} \ell\ell(\lambda \mid -) \right]$

is

$$\begin{array}{ccc} & \mu & \sigma \\ H_{(\mu,\sigma)} = \begin{array}{c} \mu \\ \sigma \end{array} & \multicolumn{2}{c}{\begin{bmatrix} \frac{\partial^2}{\partial \mu^2} \ell\ell & \frac{\partial^2}{\partial \mu \partial \sigma} \ell\ell \\ \frac{\partial^2}{\partial \sigma \partial \mu} \ell\ell & \frac{\partial^2}{\partial \sigma^2} \ell\ell \end{bmatrix}} \end{array}$$

$\boxed{Q:}$ What do we do to get $\hat{I}_{(\hat{\mu},\hat{\sigma})}$ from $H_{(\mu,\sigma)}$?

$\boxed{A:}$ evaluate $H_{(\mu,\sigma)}$ at $(\hat{\mu},\hat{\sigma})$ & multiply by -1

$$\hat{\Sigma}_{(\hat{\mu},\hat{\sigma})} = \left[\hat{I}_{(\hat{\mu},\hat{\sigma})} \right]^{-1} \text{ matrix inverse}$$

$$\hat{V}(\hat{\mu}_{MLE}) = \sqrt{\operatorname{diag}\left(\hat{\Sigma}_{(\hat{\mu},\hat{\sigma})} \right) [1]}$$

$$\text{Fixed effects model} = \left\{ \begin{array}{c} \text{random effects model} \\ \text{with } \sigma = 0 \end{array} \right\}$$