$\underset{\sim}{\theta} = (\theta_1, \ldots, \theta_k)$ | $\hat{\theta}_{MLE} \dot{\sim} ?$ | THT$_2$ 1(D) | STAT 206 6 Mar 21

DD extra OH

In regular parametric problems, in which

$D = \underset{\sim}{y} = (y_1, \ldots, y_n)$ support (Y_i) does not depend on $\underset{\sim}{\theta}$, ①

$$\hat{\theta}_{MLE} \underset{RS}{\dot{\sim}} N_k\left(\underset{\sim}{\theta}, \hat{\Sigma}_{\hat{\theta}_{MLE}}\right) \text{ as } n \to \infty \text{ with } k \text{ fixed}$$

in which $\hat{\Sigma}_{\hat{\theta}_{MLE}} = \hat{I}^{-1}_{\hat{\theta}_{MLE}}$ and

$$\hat{I}_{\hat{\theta}_{MLE}} = -\left(H \Big|_{\theta = \hat{\theta}_{MLE}} \right)$$

Hessian of log likelihood function

$\text{bias}(\hat{\theta}_{MLE}) \to 0$ as $n \to \infty$

bias ex. for $\hat{\theta}_{MLE}$

$$(Y_i \mid \mu\, \sigma\, [SM:N]\, B) \overset{IID}{\sim} N(\mu, \sigma^2) \quad (i = 1, \dots, n)$$

$$\theta = (\mu, \sigma^2)$$

$$\hat{\mu}_{MLE} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$E_{RS}(\hat{\mu}_{MLE}) = \mu$$

$y = (y_1, \dots, y_n)$

$Y = (Y_1, \dots, Y_n)$

so $\hat{\mu}_{MLE}$ is unbiased for μ

but standard estimate of σ^2 is

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$\hat{\sigma}^2_{MLE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

s^2 is ~~[illegible]~~ preferred over $\hat{\sigma}^2_{MLE}$ because

s^2 is unbiased for σ^2 & $\hat{\sigma}^2_{MLE}$ is not

$$\text{bias}(\hat{\sigma}^2_{MLE}) \doteq E_{RS}(\hat{\sigma}^2_{MLE}) - \sigma^2 < 0$$

$$\frac{(n-1)}{n} s^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2 = \frac{\cancel{n}\, \hat{\sigma}^2_{MLE}}{n}$$

③

$$\text{bias}(\hat{\sigma}^2_{MLE}) = \cancel{\theta} E_{RS}\left(\frac{n-1}{n} s^2\right) - \sigma^2$$

$$= \cancel{\theta}\ \frac{n-1}{n} \boxed{E_{RS}(s^2)}_{\sigma^2} \cancel{= \sigma^2 - \frac{n-1}{n}\sigma^2}$$

$$\cancel{= \sigma^2\left(1 - \frac{n-1}{n}\right) = \sigma^2\left(n -\right.}$$

$$= \frac{n-1}{n}\sigma^2 - \sigma^2 = \sigma^2\left(\frac{n-1}{n} - 1\right)$$

$$= -\frac{\sigma^2}{n} = O\left(\frac{1}{n}\right)$$

$$\hat{\underset{\sim}{\theta}}_{MLE} = (\hat{\mu}_{MLE}, \hat{\sigma}^2_{MLE})$$

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$k=3$

$$\ell\ell_{c}(\underset{\sim}{\theta} \mid \underset{\sim}{n}\ \mathcal{B}) = n_1 \log \theta_1 + n_2 \log \theta_2 + n_3 \log \theta_3$$

$$\underset{\sim}{\theta} = (\theta_1, \theta_2, \theta_3)$$

$$\theta_1 + \theta_2 + \theta_3 = 1 \rightarrow \theta_3 = 1 - \theta_1 - \theta_2$$

$$\ell\ell_{u}(\underset{\sim}{\theta} \mid \underset{\sim}{n}\ \mathcal{B}) = n_1 \log \theta_1 + n_2 \log \theta_2 + n_3 \log(1 - \theta_1 - \theta_2)$$