THT 1 2(B)(ii)(b) | Mr. Fisher's MLE algorithm | STAT 206 8 Feb 21

DD OH

① marginal PDF of Y_i: (for $\lambda > 0$)

$$p(y_i \mid \lambda \, E \, \mathcal{B}) = \frac{1}{\lambda} \exp\left(-\frac{y_i}{\lambda}\right) I(y_i > 0) \quad ①$$

$y = (y_1, \dots, y_n)$

② joint PDF of $\underset{\sim}{Y} = (Y_1, \dots, Y_n)$:

$$p(y \mid \lambda \, E \, \mathcal{B}) = \prod_{i=1}^{n} p(y_i \mid \lambda \, E \, \mathcal{B})$$

$$= \prod_{i=1}^{n} \frac{1}{\lambda} \exp\left(-\frac{y_i}{\lambda}\right) I(y_i > 0)$$

$I(\cdot\cdot)$ assume $= 1$

$$= \boxed{\lambda^{-n} \exp\left(-\frac{1}{\lambda} \sum_{i=1}^{n} y_i\right)} \, \boxed{I(\text{all } y_i > 0)}$$

*

$$= \left[\frac{1}{\lambda} \exp\left(-\frac{y_1}{\lambda}\right)\right] \left[\frac{1}{\lambda} \exp\left(-\frac{y_2}{\lambda}\right)\right] \cdot \dots \left[\frac{1}{\lambda} \exp\left(-\frac{y_n}{\lambda}\right)\right]$$

$s = \sum_{i=1}^{n} y_i$

$$p(\underset{\sim}{y} \mid \lambda \, \mathbb{E} \, \mathcal{B}) = \lambda^{-n} \exp\left(-\frac{s}{\lambda}\right)$$

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(3) $$\ell(\lambda \mid \underset{\sim}{y} \, \mathbb{E} \, \mathcal{B}) = c \, \lambda^{-n} \exp\left(-\frac{s}{\lambda}\right)$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \frac{s}{n}$$

$$s = n\bar{y}$$

$$= c \, \lambda^{-n} \exp\left(-\frac{n\bar{y}}{\lambda}\right)$$

$\underset{\sim}{\theta}$ = vector of unknowns $= (\lambda)$

$\dim(\underset{\sim}{\theta}) = k = 1$

in nice problems

dim(minimal sufficient statistic) ← MSS

$= k = 1$

here s and $\bar{y}$ are both MSSs

Bayesian

(4)

$p(\lambda \mid \underset{\sim}{y} \, \mathbb{E} \, \mathcal{B}) =$

$c \, p(\lambda \mid \mathcal{B}) \cdot \ell(\lambda \mid \underset{\sim}{y} \, \mathbb{E} \, \mathcal{B})$

(Mr. Fisher) (4)

$\ell\ell(\lambda \mid \underset{\sim}{y} \, \mathbb{E} \, \mathcal{B}) = -n \log \lambda - \frac{n\bar{y}}{\lambda}$

3

$$p(\lambda \mid \underline{x}\, \mathbb{F}\, \mathcal{B}) = c \; \boxed{p(\lambda \mid \mathcal{B})} \; \boxed{\cancel{\times}\; \lambda^{-n} \exp\left(-\frac{n\bar{y}}{\lambda}\right)}$$

$c_x \cdot c_* = c_*$

($c > 0$)

5 see if a <u>conjugate prior</u> exists with this sampling model/likelihood PDF

$p(\lambda \mid \mathcal{B})$ is conjugate to the ~~Exponential~~ likelihood $\boxed{\lambda^{-n} \exp\left(-\frac{n\bar{y}}{\lambda}\right)}$

unnormalized

iff

$\boxed{1}$ does this PDF match a known family of parametric PDFs?

$\boxed{2}$ is the product of 2 such PDFs in $\boxed{1}$ <u>another</u> PDF of the same form?

Gelman et al. Appendix A

Examples of parametric families of PDFs

① Beta(α, β) : PDF $\boxed{c}\, \theta^{\alpha-1}(1-\theta)^{\beta-1} \cdot I(0<\theta<1)$

dim $k=2$

$(\alpha>0, \beta>0)$

② Normal(μ, σ^2) : $\left(\begin{matrix}-\infty<\mu<+\infty \\ \sigma^2>0\end{matrix}\right)$

PDF $\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\theta-\mu}{\sigma}\right)^2\right] I(-\infty<\theta<+\infty)$

dim $2=k$

③ Exponential(λ) : $(\lambda>0)$

$\frac{1}{\lambda} \exp\left(-\frac{\theta}{\lambda}\right) I(\theta>0)$

. . .

dim $k=1$

$p(\lambda \mid \mathcal{B})$

$p(\lambda \mid y\, \mathbb{E}\, \mathcal{B}) = c\, \boxed{\lambda^{-(\alpha+1)} \exp\left(-\frac{\beta}{\lambda}\right)} \cdot$

s

(*) $\boxed{\lambda^{-n} \exp\left(-\frac{n\bar{y}}{\lambda}\right)}$

0

$$\circledast \; p(\lambda \mid x \, \mathrm{EB}) = c \, \lambda^{-[(\alpha+n)+1]} \cdot \exp\left(-\frac{\beta+S}{\lambda}\right)$$

$$= \underline{\Gamma}^{-1}\left(\underline{\alpha+n}, \; \beta+\underset{\substack{\uparrow \\ n\bar{y}}}{S}\right)$$