STAT 206
12 Jan 21

Lecture 1

Please see the wikipedia entry on "Sensitivity and Specificity"

warning: using conventional definitions, p. 17 top of the course lecture notes is wrong: there are two reasonable ways to define the false positive rate, and the official definition seems counter-intuitive to me but we need to use it:

what test says	truth: +	truth: −
+	true + TP	false + FP
−	false − FN	true − TN

official false positive rate:

$$FPR = \frac{FP}{FP + TN}$$

"incorrect" definition on p. 17 lecture notes

$$\text{"}FPR\text{"} = \frac{FP}{TP + FP}$$

[what is computed on p. 17 of the lecture notes is officially called the false discovery rate]

$$\text{FDR} = \frac{FP}{FP + TP}$$

official $$\text{FPR} = \frac{594}{594 + 98{,}406} = 0.006 = 0.6\%$$

truth

		really ⊕ $\theta = 1$	really ⊖ $\theta = 0$	
test says	⊕ $y_1 = 1$	999	594	1,593
	⊖ $y_1 = 0$	1	98,406	98,407
		1,000	99,000	100,000

official $$\text{FDR} = \frac{594}{999 + 594} \doteq 0.373 \doteq 37.3\%$$

the false negative rate (FNR) definition on p. 18 of the lecture notes is also nonstandard:

$$\text{official FNR} = \frac{FN}{FN + TP} = \frac{1}{1 + 999} = 0.001 = 0.1\%$$

what's called FNR on p. 18 is false omission rate

officially $$\text{FOR} = \frac{FN}{FN + TN} = \frac{1}{1 + 98{,}406} \doteq 1.0 \cdot 10^{-5}$$

information flow: directly

time →		
A	$A \mid B$	$A \mid BC$
$p(A) = ?$	$p(A \mid B) = ?$	$p(A \mid BC) = ?$
	B arrives	C arriving

we know from Bayes's theorem that

$$p(A \mid B) = \frac{p(A)\, p(B \mid A)}{p(B)} \quad (*)$$

what we showed in the corrected version of the 7 Jan 21 lecture was

$$p(A \mid BC) = \frac{p(A \mid B)\, p(C \mid AB)}{p(C \mid B)} \quad (**)$$

in going from $(*)$ to $(**)$, B gets moved to the other side of the conditioning bar and C takes its place

(1 = test says +, 0 not −)

$$
\begin{array}{cc}
\text{what test says} & \text{truth} \\
\left(\begin{array}{c} 0 \\ 0 \\ 1 \\ \vdots \\ \vdots \end{array}\right. & \left.\begin{array}{c} 0 \\ 0 \\ 1 \\ \vdots \\ \vdots \end{array}\right)
\end{array}
$$

(1 = really HIV, 0 not)

1 row for each subject

$n = 100{,}000$

→ sort

$$
\left[\begin{array}{c|c}
0 & 0 \\
\vdots & \vdots \\
0 & 0 \\
\hline
0 & 1 \\
\vdots & \vdots \\
0 & 1 \\
\hline
1 & 0 \\
\vdots & \vdots \\
1 & 0 \\
\hline
1 & 1 \\
\vdots & \vdots \\
1 & 1
\end{array}\right]
\begin{array}{l}
\updownarrow n_{00} \\ \\ \\ n_{01} \\ \\ \\ n_{10} \\ \\ \\ n_{11} \\ \\ \\
\end{array}
$$

4

1 column for each variable

truth

sens.

what test says	$\theta = 1$	$\theta = 0$	
$y_1 = 1$	999	594	1,593
$y_1 = 0$	1	98,406	98,407
	1000	99,000	100,000

prevalence

prev. $P(\theta = 1 \mid \mathcal{B}) = 0.01$

sens. $P(y_1 = 1 \mid \theta = 1) = 0.999$

specificity $= P(y_1 = 0 \mid \theta = 0) = 0.994$

$$P(\theta = 1 \mid y_1 = 1, \mathcal{B}) = \frac{999}{1,593} \doteq 0.627 \doteq 62.7\% \;(!)$$

$$P(y_1 = 1 \mid \mathcal{B}) = \frac{1,593}{100,000}$$

truth

test says	$\theta = 1$	$\theta = 0$
$y_1 = 1$	✓ test right	FP mistakes
$y_1 = 0$	FN mistakes	✓ test right

real-world consequences of FP | bad | doctor says Bob HIV+ when actually HIV-

['real-world consequences of FN

Bob told HIV⊖ when really HIV⊕ (really bad)

Bayesian decision theory

$(\boxed{\theta}, D, \mathcal{B}) \leftarrow P = (Q, C)$

$\downarrow$

$[a \mid \mathcal{B}), \underline{\underline{U(a, \theta \mid \mathcal{B})}}]$

action space

utility function

$U(a, \theta \mid \mathcal{B}) \in \mathbb{R}$

convention: large U better than small

several actors in this drama:

⊛ Bob

⊛ you (Bob's doctor)

⊛ Bob's health care system ($)

⊛⊛ company making Determine test

⊛ FDA

one big problem: all actors in this drama may have different utility functions

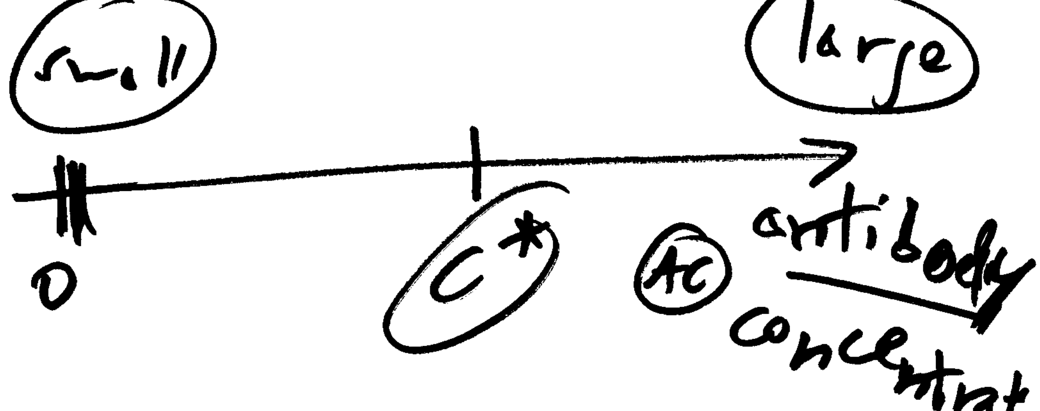
only one actor → thm: if optimal action a^*_t $(a \mid B)$

maximizes $E_{(\theta \mid D\, B)}\left[U(a, \theta \mid B) \right]$

unknown

can't do that

possible actions for company → change c^*



test says ⊕ iff AC $> c^*$

as $c^* \uparrow$ FP $\downarrow$ FN $\uparrow$

$c^* \downarrow$ FP $\uparrow$ FN $\downarrow$

MEU: maximize <u>expected</u> utility

with multiple actors in drama,

MEU with a single utility function only works when all actors agree on a common specification of $U(a, \theta \mid \underline{\underline{B}})$

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when they don't agree (everyone is in a situation of partial competition and partial cooperation with everyone else): MEU doesn't work $\boxed{*}$

in situation $\boxed{*}$ the current best math is <u>Bayesian game theory</u>