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STAT 206
2 Feb 21
Lecture ①

Quiz 2 case study:

$\underset{\sim}{y} = (y_1, \dots, y_n)$

$y_i = \begin{cases} 1 & \text{if 'yes'} \\ 0 & \text{if 'not yes'} \end{cases}$

$\underset{\sim}{Y} = (Y_1, \dots, Y_n)$:

$(Y_i \mid \theta \, \mathcal{B}) \overset{IID}{\sim} \text{Bernoulli}(\theta)$ $(i = 1, \dots, n)$ (*)

pop. mean of 1s & 0s (pop. rate of self-reported full employment in SC in mid-Jan 2021)

this is our (probability) sampling model

$S \triangleq \sum_{i=1}^{n} Y_i$

$(S \mid \theta \, \mathcal{B}) \sim$ Binomial(θ)

joint (sampling) dist. (PMF)

of $\underset{\sim}{Y}$ is $f_{\underset{\sim}{Y}}(\underbrace{y_1, \dots, y_n}_{\underset{\sim}{y}} \mid \theta \mathcal{B})$

vector

$$= p_{\underset{\sim}{Y}}(\underset{\sim}{y} \mid \theta \mathcal{B}) =$$

$y_i \in \{0, 1\}$

and　n-dimensional PMF

$$P(Y_1 = y_1, Y_2 = y_2, \dots, Y_n = y_n \mid \theta \mathcal{B})$$

IID　1-dimensional PDFs

$$= P(Y_1 = y_1 \mid \theta \mathcal{B}) \cdot P(Y_2 = y_2 \mid \theta \mathcal{B})$$

$$\cdots P(Y_n = y_n \mid \theta \mathcal{B})$$

$$= \prod_{i=1}^{n} P(Y_i = y_i \mid \theta \mathcal{B})$$

$$P(Y_i = y_i \mid \theta \mathcal{B}) = \begin{cases} \theta & \text{if } y_i = 1 \\ 1-\theta & \text{if } y_i = 0 \\ 0 & \text{otherwise} \end{cases}$$

computer science (if then, else)

$$p(\mathcal{Y}_i = y_i \mid \theta \mathcal{B}) = \theta^{y_i}(1-\theta)^{1-y_i} \cdot I(y_i \in \{0, 1\})$$

$$I(A) = \begin{cases} 1 & \text{if } A \text{ is true} \\ 0 & \text{— false} \end{cases}$$

T/F proposition

joint

$$p_{\underset{\sim}{\mathcal{Y}}}(y \mid \theta \mathcal{B}) =$$

$$\underbrace{\prod_{i=1}^{n} p(\mathcal{Y}_i = y_i \mid \theta \mathcal{B})}_{\text{product of marginals}} = \prod_{i=1}^{n} \theta^{y_i}(1-\theta)^{1-y_i} \cdot I[y_i \in \{0, 1\}]$$

$$= \theta^{y_1}(1-\theta)^{1-y_1} I(y_1 = 0 \text{ or } 1)$$

$$\cdot \theta^{y_2}(1-\theta)^{1-y_2} I(y_2 = 0 \text{ or } 1) \cdot$$

$$\cdots \theta^{y_n}(1-\theta)^{1-y_n} I(y_n = 0 \text{ or } 1)$$

$$= \theta^{y_1 + \cdots + y_n}(1-\theta)^{n-(y_1 + \cdots + y_n)} I(\text{all } y_i = 0 \text{ or } 1)$$

$$s = \sum_{i=1}^{n} y_i$$

$$p_{\underset{\sim}{Y}}(y \mid \theta\, \mathcal{B}) = \theta^s \cdot (1-\theta)^{n-s} \cdot \boxed{I(\text{all } y_i \text{ are } 0 \text{ or } 1)}$$

in this case study this indicator function is 1 because all of our y_i <u>are</u> 0 or 1

~~joint PMF~~

$$p_{\underset{\sim}{Y}}(y \mid \theta\, \mathcal{B}) = \theta^s (1-\theta)^{n-s}$$

for $y \in$ support of $\underset{\sim}{Y}$

the support of a random variable X is the set of x such that x is a possible value of X

Laplace (1780)
Fisher (1922 15)

$s = 830$
$n = 921$
quiz 2

you get ~~ℓ~~ (9) likelihood function by

(A) writing down joint PMF/PDF (5)

$p_{\underset{\sim}{X}}(\underset{\sim}{y} \mid \theta \mathcal{B})$ & (B) think of it

as a function of θ for fixed

$\underset{\sim}{y}$: def (9) $\ell(\theta \mid \underset{\sim}{y} \mathcal{B}) = \ell_c(\theta \mid \underset{\sim}{y} \mathcal{B})$

likelihood function (ell)

(C) $\cdot\, p_{\underset{\sim}{X}}(\underset{\sim}{y} \mid \theta \mathcal{B})$

$(c > 0)$

here: $\ell(\theta \mid \underset{\sim}{y} \mathcal{B}) = \theta^{s} (1-\theta)^{n-s}$

for $\theta \in (0,1)$ $= \theta^{830} (1-\theta)^{\overset{91}{\overbrace{921-830}}}$

$\hat{\Theta}_{MLE} = \dfrac{S}{n}$ r.v. estimator

$\hat{\theta}_{MLE} = s/n$ constant estimate

$$\ell(\theta \mid y\, B) = \theta^s (1-\theta)^{n-s}$$ (6)

$$\ell\ell(\theta \mid y\, B) \triangleq \log \ell(\theta \mid y\, B)$$

log likelihood function

$$= \log\left[\theta^s (1-\theta)^{n-s}\right]$$

$$= \log \theta^s + \log (1-\theta)^{n-s}$$

$$= s \cdot \log \theta + (n-s) \log (1-\theta)$$

~~the loglik f'n is locally quadratic around its max~~

$\ell(\theta \mid y\, B) \sim N(\cdot\,,\cdot)$ PDF for θ

$$= c_1 e^{-c_2(\theta - c_3)^2} \quad \cap$$

$$\ell\ell(\theta \mid y\, B) = \log c_1 - c_2 (\theta - c_3)^2$$

concave (quadratic)

= bowl-shaped-down parabola