Mr. Neyman's large-n $100(1-\alpha)\%$ confidence interval (CI) for a proportion $0 < \theta < 1$, based on an IID Bernoulli(θ) sample $\underset{\sim}{Y} = (Y_1, \ldots, Y_n)$: (discrete binary)

① compute $\hat{\theta}_n = \frac{1}{n} \sum_{i=1}^{n} Y_i$

② the approximate $100(1-\alpha)\%$ CI (with n large enough to get a good Normal approximation to the PMF of $\hat{\theta}_n$) is

$$\hat{\theta}_n \pm \underbrace{\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)} \underbrace{\sqrt{\frac{\hat{\theta}_n(1-\hat{\theta}_n)}{n}}}$$

$$(\text{estimate}) \pm \left(\begin{array}{c}\text{confidence} \\ \text{number} \\ z_\alpha\end{array}\right) \cdot \left(\hat{SE}(\hat{\theta}_n)\right)$$

Note that in this situation the population SD estimate is $\hat{\sigma}_n = \sqrt{\hat{\theta}_n(1-\hat{\theta}_n)}$

STAT 206
26 JAN 21
(Lecture)
①

$$\frac{\bar{A}-\bar{B}}{\bar{B}} = \frac{-18.6}{186} \doteq -10\%$$

average improvement

stat sig? let's see

pract. sig? yes ✓

pop

all adults similar to sampled people in all relevant ways

sample

the observed hyp. people

repeated sampling

all possible $\bar{Y}_n$ values

quant. cont. diff. $D = (B - A)$

diff. $D = (B - A)$

actual like IID = SRS

$N = ?$

$Y_1, \ldots, Y_n$ $n = 12$

$\bar{Y}_n$

$\bar{Y}_1 \ldots \bar{Y}_n$

18.6, 19.4, ... $M \to \infty$

mean $\bar{Y}_n = 18.6$ mm Hg

SD $S_D = 10.1$

mean $\Delta = ?$

SD $\sigma_D = ? < \infty$

pop PDF

hyp. IID

qqplot sample PDF

long run mean

$E_{IID}(\bar{Y}_n) = \Delta$

est. long run SD

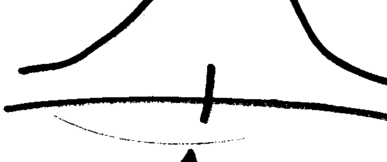
$\hat{SE}(\bar{Y}_n) = \frac{S_D}{\sqrt{n}} \doteq 2.9$

$n = 12$

mean $\bar{Y}_n = ?$ (ex. 19.4)

s.t. to (*) 10.5

$\hat{SE}$ 2.9



PDF of $\bar{Y}_n$

Δ

Good D.S. rule: Check pract. sig. first; if a diff. is not pract. sig., don't bother to check stat. sig.

The ~~ideal~~ actual population to which we can correctly generalize inferentially is {all subjects similar to the sampled subjects in all relevant ways}

goal: broadest scope of valid generalizability inferentially in generalizing outward from your data

inferential summary

pop.	unknown pop. summary of main interest	Δ = pop. mean improvement under captopril (SBP)
sample	best estimate of Δ internal to data	$\bar{Y}_n = 18.6$ mmHg $= \hat{\Delta}_n$
repeated sampling	give or take for $\bar{Y}_n$ as est. of Δ	$\hat{SE}(\bar{Y}_n) \doteq 2.9$ mmHg
	99.9% CI for Δ	$(5.6, 31.5)$ mmHg

under the modeling assumptions here, we think that Δ is around 19 mmHg (realized value $\hat{\Delta}_n = \bar{Y}_n$), give or take about $\hat{SE}(\bar{Y}_n) = 2.9$ mmHg, and a 99.9% CI for Δ runs from 5.6 mmHg to 31.5 mmHg

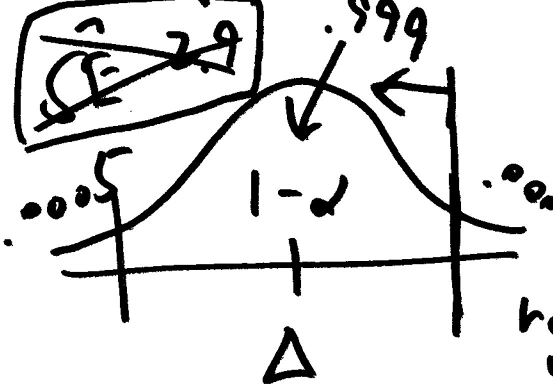
$$E_{IID}(\bar{D}_n) = \Delta$$

STAT 131

$$\widehat{SE}_{IID}(\bar{D}_n) \hat{=} \sqrt{\hat{V}_{IID}(\bar{D}_n)} = \frac{\hat{\sigma}_D}{\sqrt{n}} = \frac{s_D}{\sqrt{n}} = 2.91 \text{ mmHg}$$

SE $\frac{\sigma_D}{\sqrt{n}}$

$\boxed{\cancel{\widehat{SE} \; 2.9}}$



(*) PDF of $\bar{D}_n$

raw units (mmHg)

$-\Phi^{-1}(1-\frac{\alpha}{2})$ $\quad \Phi^{-1}(1-\frac{\alpha}{2})$ standard units (z)

$-3.29 = -z_\alpha$

qnorm(.9995)

$+3.29 = z_\alpha$

99.9% CI

↓

$100(1-\alpha)\%$

$\alpha = .001$

$\frac{\alpha}{2} = .0005$

$100(1-\alpha)\%$ CI for Δ is

$$\hat{\Delta}_n \pm \underbrace{\Phi^{-1}\left(1-\frac{\alpha}{2}\right)}_{z_\alpha} \cdot \underbrace{\boxed{\frac{\sigma_D}{\sqrt{n}}}}_{SE(\hat{\Delta}_n)}$$

($\bar{D}_n$)

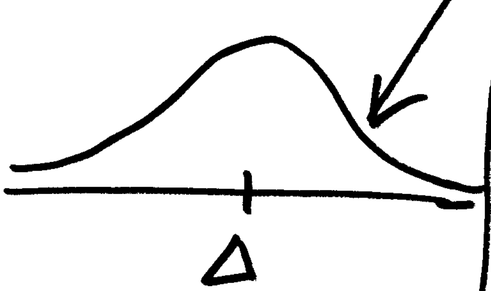
(1908)

Gosset

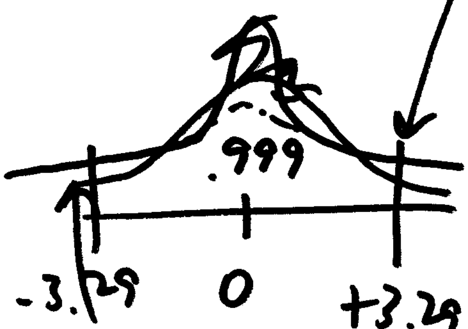
Guiness?

brewery

standard normal



PDF of $\bar{D}_n$, accounting for uncertainty about σ_D



PDF of $\dfrac{\bar{D}_n - \Delta}{s_D/\sqrt{n}}$, assuming pop PDF is Normal

t_{n-1} PDF

degrees of freedom

as $n \to \infty$

t_n PDF $\to$ standard normal PDF

actual $100(1-\alpha)\%$ CI for Δ, accounting for uncertainty in σ_D:

$$\hat{\Delta}_n \pm t^{-1}_{n-1}\left(1-\frac{\alpha}{2}\right)\cdot\frac{s_D}{\sqrt{n}}$$

$$18.6 \pm 4.44 \cdot \frac{10.1}{\sqrt{12}} = (5.6,\ 31.5) \text{ mmHg}$$

7

(n-1) degrees (df) of freedom?

free y_1 ✓
free y_2 ✓
not free y_3 X

3 = n

mean 7 = $\bar{y}_n = \frac{y_1+y_2+y_3}{3}$

n obs, (n-1) df for measuring spread

we used the data once in computing $\bar{y}_n = \hat{\Delta}_n$ to estimate Δ

$$s_D = \sqrt{\frac{1}{n-1}\underbrace{\sum_{i=1}^{n}(y_i-\bar{y})^2}_{n \text{ terms}}}$$

unbiased variance estimation

$E_{IID}(s_D^2) = \sigma_D^2$

$E_{IID}(s_D) \neq \sigma_D$

but only (n-1) independent pieces of info about spread